CERN Axion Solar Telescope as a probe of large extra dimensions

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We explore the potential of the CERN Axion Solar Telescope (CAST) for testing the presence of large extra dimensions. The CAST experiment has originally been proposed to search for solar axions with a sensitivity supposed to provide a limit on the axion-photon coupling $g_{a\gamma\gamma} \lesssim 5 \times 10^{-11}\text{ GeV}^{-1}$ or even lower. The expected bound on the coupling constant is by a factor of ten more stringent than the current experimental results. This bound extends for the first time beyond the limit dictated by astrophysical considerations. As a tuning experiment planning to explore the axion mass region up to about 1 eV, CAST would also be sensitive to the existence of Kaluza-Klein massive states. Therefore, the detection of x rays at least at two pressures may be the signature of large extra dimensions. From this requirement we find that CAST may test (two) large extra dimensions with a (common) compactification radius $R$ down to around 250 nm if $m_{\text{PQ}} < 1/(2R)$, and down to around 370 nm if $1/(2R) < m_{\text{PQ}}$, where $m_{\text{PQ}}$ is the Peccei-Quinn mass.

I. INTRODUCTION

A few years ago an approach was put forward [1] in which heavy mass scales in four dimensions could be replaced by lighter mass scales in higher dimensions. Such a class of theories is nowadays conventionally considered in the context of the brane paradigm. In one class of models extra dimensions are felt only by gravity (as well as other fields transforming as singlet under the standard-model gauge group); in the other class they are felt also by gauge fields. In the former case, the standard-model fields are confined to a $(3+1)$-dimensional subspace of a higher-dimensional space some dimensions of which are compactified with a relatively large radius. The absence of any observed deviation from ordinary Newtonian gravity in Cavendish-type laboratory experiments implies that the largest compactification radius is smaller than around 0.2 mm [2].

The main goal in both classes of models is to provide a unified theory in which the electroweak scale $M_W \sim 10^2\text{ GeV}$ and the high energy scales (Planck [1], string [3], and grand unified theory (GUT) scales [4]) can coexist. The same scenario has also been successfully applied to neutrino as well as to axion phenomenology. Namely, a higher-dimensional seesaw mechanism may provide light neutrino masses without heavy mass scales [5]. Similarly, axion invisibility can be achieved in extra dimensions even with a low fundamental Peccei-Quinn (PQ) scale [6].

The CERN Axion Solar Telescope (CAST) is designed to search for solar axions of a broad energy spectrum which peaks at about 4 keV, through their conversion into real photons inside the transverse magnetic field [7,8]. This telescope may improve the current laboratory bounds on the axion-photon coupling, $g_{a\gamma\gamma} \lesssim 6 \times 10^{-10}\text{ GeV}^{-1}$ for $m \lesssim 0.03\text{ eV}$ and $g_{a\gamma\gamma} \lesssim 6.8 - 10.9 \times 10^{-10}\text{ GeV}^{-1}$ for $m \sim 0.05 - 0.27\text{ eV}$ [9], by a factor of ten or even more. It also has the potential to extend for the first time the axion searches beyond the limit $g_{a\gamma\gamma} \lesssim 10^{-10}\text{ GeV}^{-1}$ arising from astrophysical constraints on anomalous energy loss by stars [10]. Although the CAST telescope could in principle be sensitive to axion masses in the range of a few keV, the coherence-loss constraints [11,12] reduce the sensitivity down to around 1 eV.

The first goal of the present paper is to interpret prospects of CAST in the light of the theory with large extra spatial dimensions. We focus on the case when the limit on the size of two large extra compact dimensions is set by direct tests of gravity [2]. Our second goal is to explore the potential of CAST for testing the presence of large extra dimensions.

II. QCD AXIONS AND CAST

Axions are pseudoscalars arising in models which resolve the strong CP problem in quantum chromodynamics (QCD) by the PQ mechanism [13]. Owing to their potential abundance in the early universe, they are also well-motivated candidates for the dark matter of the universe. In both classes of (conventional) invisible axion models referred to as Kim-Shifman-Vainshtein-Zakharov (KSVZ) or hadronic axion models [14] (where axions do not couple to electrons at tree level) and Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) or grand unified theory (GUT) models [15], the axion-photon coupling strength is given by the relation

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_{\text{PQ}}} (E/N - 1.93 \pm 0.08).$$

Here $E/N$ is a model-dependent numerical parameter for hadronic axions, while for DFSZ axions $E/N = 8/3$. Furthermore, the mass of the (QCD) axion $m_{\text{PQ}}$ is related to the PQ symmetry breaking scale $f_{\text{PQ}}$ by
\[ m_{\text{PQ}} = 6 \text{ eV} \sqrt[10]{\frac{10^6}{1 \text{ GeV}}}. \]  

In order to avoid ambiguities owing to the model-dependence of the parameter \( E/N \) for hadronic axions, it proved more convenient to make constraints on the axion-photon coupling than on the PQ energy scale or on the axion mass.

In contrast, cosmological considerations and astrophysical arguments (i.e., axion emission due to nucleon-nucleon bremsstrahlung from the supernova SN 1987A) bound the axion mass into two possible ranges [16]. The first window is \( 10^{-5} \text{ eV} \lesssim m_{\text{PQ}} \lesssim 10^{-2} \text{ eV} \), in which case the axion could constitute the cold dark matter of the universe. The second one, being around ten to twenty electronvolts, appears to be of interest for hot dark matter. However, such astrophysical constraints on \( m_{\text{PQ}} \), although the most stringent, suffer from statistical weakness (with only 19 neutrinos being observed) as well as from all uncertainties related to the axion emission from a hot/dense medium. It is therefore of crucial importance to probe the axion properties in a model-independent way [17,18].

Currently, laboratory searches for solar axions [9,12,19,20] are being extended by the CAST experiment at CERN. This telescope uses a decommissioned Large Hadron Collider (LHC) prototype magnet with a field of 9 T and a length \( L = 10 \text{ m} \). The magnet contains two straight beam pipes with an effective cross sectional area \( S = 2 \times 14 \text{ cm}^2 \), and is mounted on a moving platform with low-background x-ray detectors on either end allowing it to track the Sun about 3 hours per day.

Hadronic axions could be produced abundantly in the core of the Sun by the Primakoff conversion of the blackbody photons in the Coulomb fields of nuclei and electrons in the solar plasma. The outgoing axion flux is robust and does not depend on subtle details of the solar model. It is approximatively given by [21]

\[
\frac{d\Phi(E)}{dE} = 4.20 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1} \left( \frac{8 s_{\gamma\gamma}}{10^{-10} \text{ GeV}^{-1}} \right)^2 
\times \frac{E_p^2}{e^{E/1.1} - 0.7} \left( 1 + 0.02m \right).
\]  

Here \( d\Phi(E)/dE \) is the axion flux at the Earth, differential with respect to axion energy \( (E) \), and expressed as a function of axion mass \( (m) \). The quantities \( E, p = \sqrt{E^2 - m^2} \), and \( m \) are to be taken in keV. The probability for an axion-to-photon conversion in the presence of a transverse magnetic field \( (B) \) and a refractive medium \( (i) \) is given by [11]

\[
P_{\mu_{\gamma}}(m) = \left( \frac{B s_{\gamma\gamma}}{2} \right)^2 \frac{1}{q_i^2 + \mu_i^2/4} \times \left[ 1 + e^{-\mu_i L} - 2e^{-\mu_i L/2} \cos(q_i L) \right],
\]  

where \( q_i = (m_i^2 - m^2)/2E \) is the momentum difference between photons in the medium and axions, and \( \mu_i \) denotes the inverse absorption length for x rays. The effective mass (plasma frequency) for an x ray in He can be described in terms of the operating pressure \( P_i \) (at 300 K) as \( m_{\text{PQ}}/1 \text{ eV} \approx (P_i/1 \text{ atm})/15 \). The coherence condition \( q_i L < \pi \) [12] requires \( m \gtrsim 0.02 \text{ eV} \) for a photon energy of 4.2 keV (the average axion energy) and a coherence length of 10 m in vacuum. To search for axions more massive, coherence can be restored by filling the magnetic conversion region with buffer gas. Integrating over all axion energies, the expected number of photons \( N_{\gamma_{iS}}(m) \), being detected during the times of solar alignment with the magnet \( (t_i) \), is finally

\[
N_{\gamma_{iS}}(m) = \int_0^\infty \frac{d\Phi(E)}{dE} P_{\mu_{\gamma}}(m) S_i dE,
\]  

assuming 100% detection efficiency for the conversion x rays. At a fixed pressure \( P_i \), the response of CAST will be a sharply peaked function of the actual axion mass \( m \), with the fractional resolution \( \Delta m/m_{\gamma} \approx 5.2 \times 10^{-4} (m_{\gamma}/1 \text{ eV})^{-2} \). A general analysis of the experimental prospects [7] explores the full two-dimensional \( (m, s_{\gamma\gamma}) \) space for QCD axions rather than the narrow band defined by conventional axion models (although it remains the best-motivated region), as it is shown in Fig. 2. The experiment is being operated in a scanning mode in which the gas pressure is varied in appropriate steps (1 yr with vacuum, an additional 1 yr with a He gas pressure increased from 0–1 atm in 100 increments, and an additional 1 yr with 1–10 atm in 350 increments) to cover a range of possible axion masses up to 0.82 eV.

### III. CAST and Large Extra Dimensions

Large extra dimensions aim to stabilize the mass hierarchy (i.e., the hierarchy between the Planck scale and the electroweak scale) by producing the huggeness of the Planck mass \( M_{\text{Pl}} \) via the relation

\[
M_{\text{Pl}}^2 = M_D^{n+2} V_n,
\]  

where \( V_n = R^n \) is the full volume of the compactified space, and the fundamental scale is set at \( M_D \sim \text{TeV} \). As already stressed, a singlet higher-dimensional axion field is also free to propagate into the bulk and therefore a similar volume-suppressed formula can be used to lower the fundamental Peccei-Quinn symmetry-breaking scale \( f_{\text{PQ}} \) [1,6,22]

\[
f_{\text{PQ}}^{2} = f_{\text{PQ}}^{2} M_{S}^{\delta} V_{n},
\]  

where \( M_{S} \) is the string scale, \( M_{S} \sim M_{D} \). Since the phenomenologically allowed region for \( f_{\text{PQ}} \) (also generating the coupling between the axion and matter) is such that \( f_{\text{PQ}} \leq M_{\text{Pl}} \), the axion must be restricted to a subspace of the full higher-dimensional bulk \( (\delta < n) \), if \( f_{\text{PQ}} \) is to reside in the TeV range [6]. Still, \( \delta = n \) is possible for \( f_{\text{PQ}} \leq \text{TeV} \) [21,23].

In Ref. [6] the full generalization of the higher-dimensional PQ mechanism was given, including a thorough discussion of how extra space dimensions may contribute to the invisibility of the PQ axion. All new phenomena contributing to the invisibility of the axion and found there rely on
The results from Ref. [6] of the masses of the KK modes are given by
\[ m_i = \frac{1}{R} \sqrt{n_1^2 + n_2^2 + \cdots + n_δ^2} = \frac{\delta}{R}, \]
where we assume that all \( n \) extra dimensions are of the same size \( R \). When the mass splitting for the size \( R (\approx 1/R) \) is sufficiently small, one is allowed to use integration instead of summation [24]. We have already mentioned that because of the nontrivial axion mass matrix, neither the four-dimensional axion nor the KK states represent the mass eigenstates. Instead, the eigenvalues are given as solutions to the transcendental equation [6]
\[ \pi R \lambda \cot(\pi R \lambda) = \frac{\lambda^2}{m_{PQ}}. \]

Hence, in order to estimate the number of modes with the KK index between \( n \) and \( n + dn \), one should parametrize the whole set of eigenvalues of Eq. (9). This can be done by solving Eq. (9) for two limiting cases, \( m_{PQ}R \ll 1 \) and \( m_{PQ}R \ll 1 \). We find for the eigenvalues
\[ \lambda_0 = m_{PQ}, \]
\[ \lambda_n = \frac{n}{R}, \quad n = 1, 2, \ldots \text{ if } m_{PQ}R \ll 1, \]
and
\[ \lambda_n = \frac{2n + 1}{2R}, \quad n = 0, 1, 2, \ldots \text{ if } m_{PQ}R \gg 1. \]

The results from Ref. [6], where only the mass of the axion zero mode was estimated, can now be easily reproduced from our expressions (10) and (11). In Fig. 1 we show the mass of the first KK state as a function of \( m_{PQ}R \). It can be seen how the mass quickly approaches its limiting value \((3/2)R^{-1}\). A similar feature was found in Ref. [6] for the zero mode. Finally, the total number of x rays due to all modes of the KK tower reads
\[ N_{\gamma i}^{KK} = S_{\gamma i} R^\delta \int_0^\infty dmm^{\delta-1}N_{\gamma i}(m)G(m) \quad \text{if } m_{PQ}R \ll 1 \]
and
\[ N_{\gamma i}^{KK} = S_{\gamma i} R^\delta \int_0^\infty dmm^{\delta-1}N_{\gamma i}(m+1/R)G(m+1/R) \quad \text{if } m_{PQ}R \gg 1, \]
where \( S_{\delta}=2\pi^\delta/\Gamma(\delta/2) \) is the surface of a unit radius in \( \delta \) dimensions and \( G(m) \) is defined as
\[ G(m) = \frac{m^4}{m^2 + 1 + \pi^2/y^2} - 2, \]
with \( m = m_{PQ} \) and \( y = 1/m_{PQ}R \). The function \( G(m) \) arises from the mixing between the KK axion modes entering the KK decomposition of the higher-dimensional axion field and the corresponding normalized mass eigenstates [6]. It also implies both production and detection of KK axions to occur on our standard-model brane. The function \( G(m) \) also incorporates the effect of rapid decoherency [6] of the only linear combination of KK states of the bulk axion which couples to standard-model fields. This means that the production and subsequent detection of this particular linear combination of KK states are strongly suppressed. As a consequence our results always reflect a volume-suppressed coupling \( g_{\alpha \gamma} \sim 1/f_{\alpha} \). If it not for the decoherency, the linear combination would be coupled to photons with an unsuppressed coupling \( 1/f_{\alpha} \).

IV. DISCUSSION

In order to achieve an upper limit on the coupling of the axion to photons from the prospects of CAST in the framework of large extra dimensions, we apply the central limit theorem at 3σ level
\[ \sum_i N_{\gamma i}^{KK} \leq 3 \sqrt{\sum_i N_{bi}}, \]
where \( N_{bi} \) is the background of the x-ray detector (with numerical values taken from Ref. [7]). For the sake of simplicity, it is assumed that all axions have an average energy of 4.2 keV. Combining Eq. (15) with Eqs. (3)–(5) and (12)–(14) for the case of two extra dimensions (we take the largest compactification radius of 0.150 mm as set by direct tests of Newton's law [2]), we have derived limits on the axion-photon coupling \( g_{\alpha \gamma} \) as a function of the fundamental PQ mass, as shown in Fig. 2. Although the multiplicity of KK states to which CAST could be sensitive is large (\( \sim 10^5 \) for \( \delta=1 \) and \( \sim 10^6 \) for \( \delta=2 \)), one can see from Fig. 2 that the
The obtained limits on $g_{a\gamma\gamma}$ are shown in Fig. 2. In the regime $m_{PQ} R \geq 1$, $G(m)$ decreases as fast as $1/(m_{PQ} R)^{5}$. It is just the regime in which the obtained limits on $g_{a\gamma\gamma}$ cannot be coupled with the zero-mode axion mass $m_{a} \approx (1/2) R^{-1} = 6.6 \times 10^{-4}$ eV via relations (1) and (2) because in higher dimensions the mass of the axion is approximatively given by $m_{a} \approx \min((1/2) R^{-1}, m_{PQ})$ [6]. In contrast with the case of ordinary QCD axions, in theories with large extra dimensions zero-mode axions with masses outside the favored band (as determined by conventional axion models in four dimensions) arise quite naturally.

Now we would like to point to a new phenomenon predicted for CAST: sensitivity to particular KK axions. We have already noted that physical KK modes are given by Eqs. (10) and (11). It is expected that more than one axion signal may be observed at different pressures of the gas. Therefore, the detection of the corresponding x rays at least at two pressures may be the signal for the presence of large extra dimensions. As the CAST experiment is scanning the range of axion masses up to 0.82 eV, this requirement actually defines a sensitivity of the experiment to test the compactification radius. From Eqs. (10) and (11) we obtain $R \geq \frac{370}{m_{PQ}} \text{nm}$ if $m_{PQ} R \geq 1$ and $R \geq \frac{250}{m_{PQ}}$ if $m_{PQ} R \leq 1$, with $g_{a\gamma\gamma} \approx 10^{-10}$ GeV$^{-1}$. It should be noted here that with the CAST sensitivity to $g_{a\gamma\gamma}$ the former result holds only for $m_{PQ} R \geq 1$; for $m_{PQ} R \leq 1$ the sensitivity rapidly decreases due to the suppression from the $G(m)$ function. The present modifications of CAST may increase its sensitivity to $g_{a\gamma\gamma}$ by a factor of 1.5 [8], providing the sensitivity as mentioned above is of the order of that derived from the solar age consideration [25]. Note that in a recent review of the Particle Data Group [26] the bound on $R$ for the case of two extra dimensions, coming from astrophysics, was listed to have a value within the range 90 to 660 nm (for the most stringent constraints, see the recent work [27]).

In conclusion, we have explored the potential of the CAST experiment for observing KK axions coming from the solar interior. Because of the restrictive coherence condition, in theories with two extra dimensions (with $R \approx 0.150$ mm) a sensitivity in axion-photon coupling improves at most one order of magnitude in both data taking phases. In this case, the obtained limit on $g_{a\gamma\gamma}$ cannot be coupled with the mass of the axion, which is essentially given by the (common) radius of the extra dimensions. In addition, we have demonstrated that the CAST experiment, being a tuning experiment with respect to axion masses, may not be sensitive only to an integrated effect of KK modes up to the kinematical limit but also to particular KK axions. With a requirement to have at least two signals while changing pressure of the gas, we have found that CAST is capable of probing (two) large extra dimensions with a compactification radius $R$ down to around 250 nm if $m_{PQ} < 1/(2R)$, and down to around 370 nm if $1/(2R) < m_{PQ}$.


[25] By simply scaling the solar age limit of QCD axions ($g_{a\gamma\gamma} \approx 2.4 \times 10^{-9}$ GeV$^{-1}$) [16] with $(Rm_{\max})^{-\delta/2}$ due to the multiplicity of KK modes and observing that an allowed maximum mass is $m_{\max} \approx$ keV before the solar flux gets suppressed by the kinematic threshold, one obtains for two extra dimensions: (i) for $R = 0.150$ mm: $g_{a\gamma\gamma} \approx 3 \times 10^{-12}$ GeV$^{-1}$ if $\delta = 1$ and $g_{a\gamma\gamma} \approx 3 \times 10^{-15}$ GeV$^{-1}$ if $\delta = 2$, and (ii) for $R = 250$ nm: $g_{a\gamma\gamma} \approx 7 \times 10^{-11}$ GeV$^{-1}$ if $\delta = 1$ and $g_{a\gamma\gamma} \approx 2 \times 10^{-12}$ GeV$^{-1}$ if $\delta = 2$.